Vibrational Analysis of Locomotives

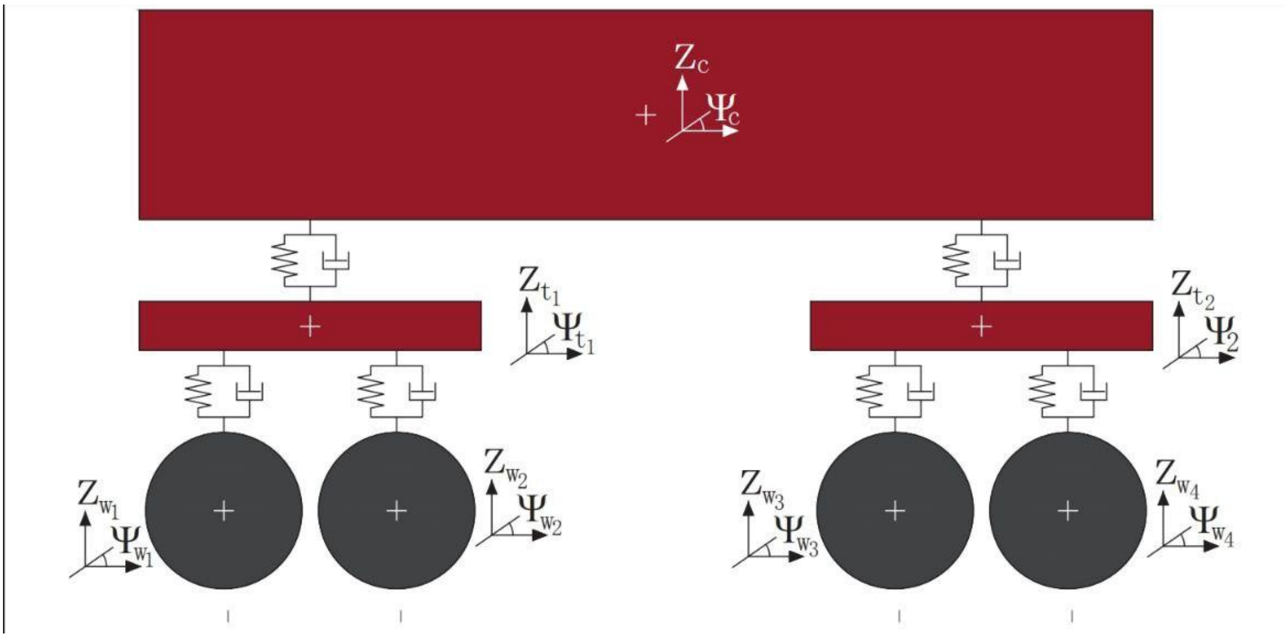
Introduction

Here's our problem statement. We here let's consider an example of locomotives. Locomotives has two crankshaft at 90 degree phase difference, and hence it has various couple and forces acting over the body, and has multiple degree of freedoms. For simplicity we consider a simple wagon-trolley design, which helps us to study the analytic effect of these couples over the wheels of the locomotives. Now a days, we use Electric motors to reduce jerk due to lesser movements, but when it comes to the efficiency, we have seen for heavy load carrying purposes, these diesel engine locomotives are more efficient.

Source: https://www.google.com/url?sa=i&url=https%3A%2F%2Fwww.progressrail.com%2Fen%2Frollingstock%2Flocomotives%2Ffreight.html&psig=AOvVaw1Gi5Fa8xZfs_Ml9sBjCHR_&ust=1596972665395000&source=images&cd=vfe&ved=0CAMQjB1qFwoTCKCo7PnAi-sCFQAAAAAdAAAAABAD

Our Model:

There are two ways to model a design:
1) Mechanical Modelling



A basic draft model

2) Mathematical Modelling

Assembeled Motion Equations for the vehicle model

The reaction forces on each masses were calculated using force balance equation, for the values of primary and secondary stiffness and damping values we reffered to the Manchester Benchmark, which is:

Vehicle specification - Benchmark Vehicle 1		
Masses and Inertias		
wheel mass	1813	kg
bogies mass	2615	kg
bogies pitch inertia	1476	$kg\,m^2$
body mass	32000	kg
body pitch inertia	1970000	$kg\,m^2$
Primary suspension		
stiffness	1220	kN/m
damping	4	kNs/m
damper series stiffness	1000	kN/m
Secondary suspension		
stiffness	1220	kN/m
damping	32	kNs/m
damper series stiffness	6000	kN/m

Before solving our mathematical model, we assume the pitch to be same for both the bogies as the effects produced by train bogies are negligible. And the length between wheel base to be 2lw, whereas the length between the pivots on the bogies is assumed to be 2lt.

First just name every damping and spring constants. The springs and damper supporting the body are called secondary suspension, and let's just name them as k2 and c2 respectively. Similarly, those springs and dampers supporting the tyres and bogies are called primary suspension, and let it be k1 and c1. Thus whole thing will now look like:

for body,

$m_c \ddot{z}_c + c_2 (\dot{z}_c - \dot{z}_{t1}) + c_2 (\dot{z}_c - \dot{z}_{t2}) + k_2 (z_c - z_{t1}) + k_2 (z_c - z_{t2}) = m_c g$
=> **$m_c \ddot{z}_c + c_2 (2\dot{z}_c - \dot{z}_{t1} - \dot{z}_{t2}) + k_2 (2z_c - z_{t1} - z_{t2}) = m_c g$** ....(1)

$J_c \psi_c(:) + c_2 (\Psi_c(.) - \Psi_{t1}(.))l_t + c_2 (\Psi_c(.) - \Psi_{t2}(.))l_t + k_2 (\Psi_c - \Psi_{t1})l_t + k_2 (\Psi_c - \Psi_{t2})l_t = 0$
=> **$J_c \psi_c(:) + c_2 (2 \Psi_c(.) - \Psi_{t1}(.) - \Psi_{t2}(.))l_t + k_2 (2\Psi_c - \Psi_{t1} - \Psi_{t2})l_t = 0$** ....(2)

For bogies,

=> **$m_{t1} \ddot{z}_{t1} + c_2 (\dot{z}_{t1} - \dot{z}_c) + c_1 (2\dot{z}_{t1} - \dot{z}_{w1} - \dot{z}_{w2}) + k_2(z_{t1} - z_c) + k_1 (2z_{t1} - z_{w1} - z_{w2}) = m_{t1} g$** ....(3)

=> **$m_{t2} \ddot{z}_{t2} + c_2 (\dot{z}_{t2} - \dot{z}_c) + c_1 (2\dot{z}_{t2} - \dot{z}_{w3} - \dot{z}_{w4}) + k_2(z_{t2} - z_c) + k_1 (2z_{t2} - z_{w3} - z_{w4}) = m_{t2} g$** ....(4)

=> **$J_{t1} \psi_{t1}(:) + c_1 (2 \Psi_{t1}(.) * l_w - \dot{z}_{w1} - \dot{z}_{w2}) + k_1 (2\Psi_{t1} * l_w - z_{w1} - z_{w2}) = 0$** ....(5)

=> **$J_{t2} \psi_{t2}(:) + c_1 (2 \Psi_{t2}(.) * l_w - \dot{z}_{w3} - \dot{z}_{w4}) + k_1 (2\Psi_{t2} * l_w - z_{w3} - z_{w4}) = 0$** ....(6)

For wheels,

For i = 1,2

=> **$m_{wi} \ddot{z}_{wi} + c_1 (\dot{z}_{wi} - \dot{z}_{t1} - \Psi_{t1}(.) * l_w) + k_1 (z_{wi} - z_{t1} - \Psi_{t1} * l_w) + R = m_{wi} * g$** ....(7-8)

For, i=3,4

=> **$m_{wi}\ddot{z}_{wi} + c_1 (\dot{z}_{wi} - \dot{z}_{t2} – \dot{\Psi}_{t2}*l_w) + k_1 (z_{wi} – z_{t2} – \Psi_{t1}*l_w) + R = m_{wi}*g$** ....(9-10)

All these eight equation turns out to be written in the matrix form:

$$M \ddot{z} + C \dot{z} + K z = F \quad ....(11)$$

Also, mw1=mw2=mw3=mw4=mw=1813 kg

mt1=mt2=mt3=mt4=mt=2615 kg

mc=1813 kg

Jt=1476 kg m^2

Jc=1970000 kg m^2

k1=1220 kN/m

c1=4 kNs/m

k2=1220 kN/m

c2=32 kNs/m

R=35.058 KN

This implies,

M =

1813	0	0	0	0	0	0	0	0	0
0	1970000	0	0	0	0	0	0	0	0
0	0	2615	0	0	0	0	0	0	0
0	0	0	1476	0	0	0	0	0	0
0	0	0	0	2615	0	0	0	0	0
0	0	0	0	0	1476	0	0	0	0
0	0	0	0	0	0	1813	0	0	0
0	0	0	0	0	0	0	1813	0	0
0	0	0	0	0	0	0	0	1813	0
0	0	0	0	0	0	0	0	0	1813

C =

64.0000	0	-32.0000	0	-32.0000	0	0	0	0	0
0	476.8000	0	-238.4000	0	-238.4000	0	0	0	0
-32.0000	0	40.0000	0	0	0	-4.0000	-4.0000	0	0
0	0	0	6.4000	0	0	-3.2000	-3.2000	0	0
-32.0000	0	0	0	40.0000	0	0	0	-4.0000	-4.0000
0	0	0	0	0	6.4000	0	0	-3.2000	-3.2000
0	0	-4.0000	-3.2000	0	0	4.0000	0	0	0
0	0	-4.0000	-3.2000	0	0	0	4.0000	0	0
0	0	0	0	-4.0000	-3.2000	0	0	4.0000	0
0	0	0	0	-4.0000	-3.2000	0	0	0	4.0000

K =

2440	0	-1220	0	-1220	0	0	0	0	0
0	18178	0	-9089	0	-9089	0	0	0	0
-1220	0	3660	0	0	0	-1220	-1220	0	0
0	0	0	1952	0	0	-976	-976	0	0
-1220	0	0	0	3660	0	0	0	-1220	-1220
0	0	0	0	0	1952	0	0	-976	-976
0	0	-1220	-976	0	0	1220	0	0	0
0	0	-1220	-976	0	0	0	1220	0	0
0	0	0	0	-1220	-976	0	0	1220	0
0	0	0	0	-1220	-976	0	0	0	1220

And, z=col(zc,Ψ c,zt1,Ψ t1,zt2,Ψ t2,zw1,zw2,zw3,zw4);
F=col(17785.53,0,25653.15,0,25653.15,0,-17272.9575,-17272.9575,-17272.9575,-17272.9575);

Natural frequency

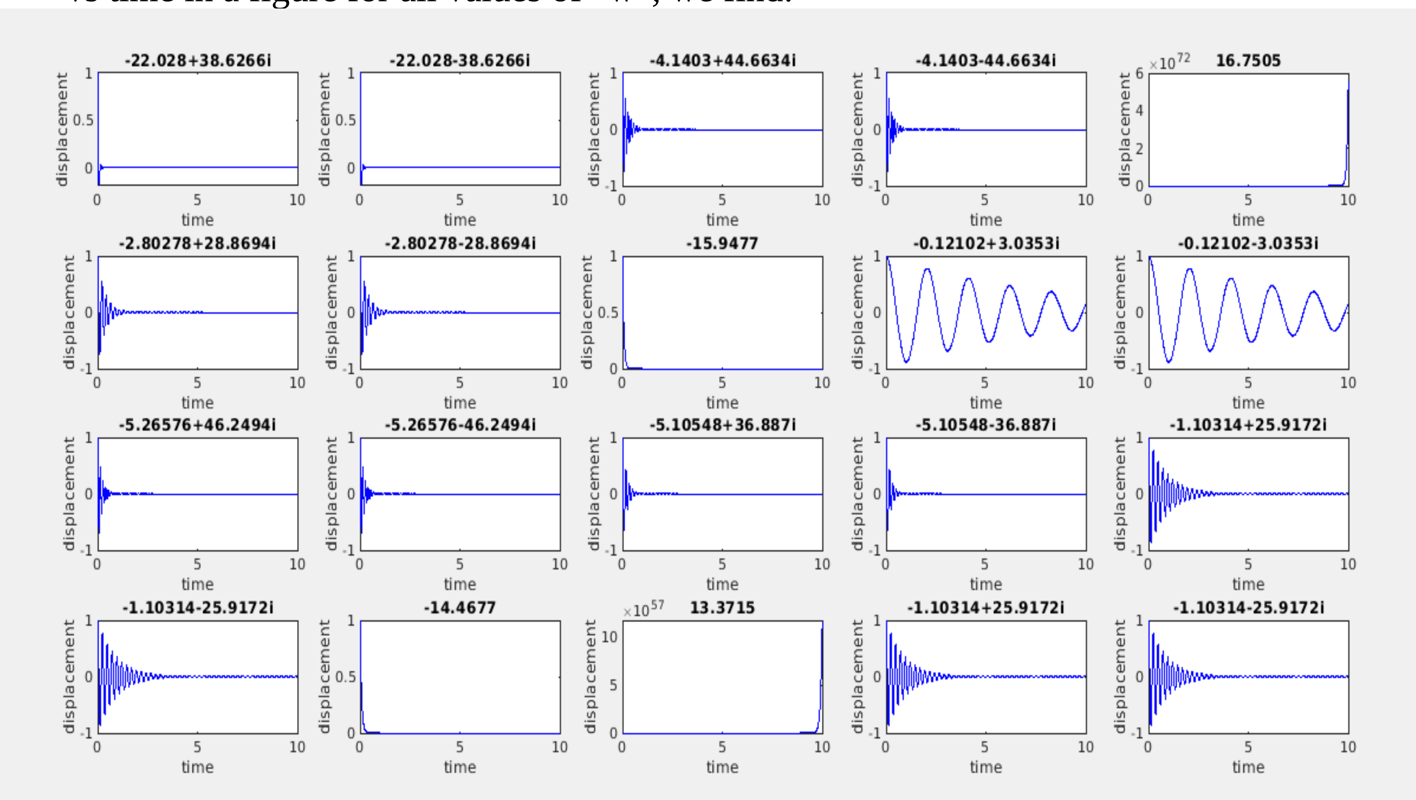
The damping factors are functions of natural frequency if the system and proportionality factors. Natural frequency are found out for homogeneous solution to determine its mode shape. I have used matlab for finding the natural frequency matrix, using [X,e,s]=polyeig(K,C,M). In which, X and e are the eigen vectors and eigenvalues pairs.

And omega(i,i)=e(i), where i varies from 1 to 20.

ω = diag(-22.0280 +38.6266i, -22.0280 -38.6266i, -4.1403 +44.6634i, -4.1403 -44.6634i, 16.7505 + 0.0000i, -2.8028 +28.8694i, -2.8028 -28.8694i, -15.9477 + 0.0000i, -0.1210 + 3.0353i, -0.1210 – 3.0353i, -5.2658 +46.2494i, -5.2658 -46.2494i, -5.1055 +36.8870i, -5.1055 -36.8870i, -1.1031 +25.9172i, -1.1031 -25.9172i, -14.4677 + 0.0000i, 13.3715 + 0.0000i, -1.1031 +25.9172i, -1.1031 -25.9172i)$_{20x20}$

Mode Shape

With omega value as known, we can plot mode shapes using mode(i)=exp(w(i,i)*time), where i varies from 1 to 20. Plotting mode displacement vs time in a figure for all values of "w", we find:



Conclusion:

The design of the locomotives is clear to be stable after some displacement given to it. As we can see very clearly from the plots formed for mode shapes in our case is damping for almost every cases after some time. Some of the eigen values shows displacement to be for gone with time, which is irrelevent. That means, this homogeneous solution gives all solution possible for our matrix Quadratic equation, which will gwt removed after matching our results with the simulated results or experimentally determined modes of vibration.

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